Trigonometry Identities

Reciprocal Identities:

$$\sin x = \frac{1}{\csc x},\ \cos x = \frac{1}{\sec x},\ \tan x = \frac{1}{\cot x}$$

Ratio Identities:

$$\tan x = \frac{\sin x}{\cos x},\ \tan x = \frac{\sec x}{\csc x},\ \cot x = \frac{\cos x}{\sin x},\ \cot x = \frac{\csc x}{\sec x}$$

Pythagorean Identities:

$$\sin^2 x + \cos^2 x = 1,\ \sec^2 x - \tan^2 x = 1,\ \csc^2 x - \cot^2 x = 1$$

Negative Identities:

$$\cos(-x) = \cos x$$
$$\sin(-x) = -\sin x$$
$$\tan(-x) = -\tan x$$

Co-Function Identities:

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$
$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$
$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

Sum/Difference Identities:

$$\sin(x+y) = \sin x\cos y + \cos x\sin y$$
$$\sin(x-y) = \sin x\cos y - \cos x\sin y$$
$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$
$$\cos(x-y) = \cos x\cos y + \sin x\sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$$

****Double Angle Identities:**

$$\sin(2x) = 2\sin x\cos x$$
$$\cos(2x) = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$$

****Half Angle Identities:**

$$\sin\left(\frac{1}{2}x\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$
$$\cos\left(\frac{1}{2}x\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$
$$\tan\left(\frac{1}{2}x\right) = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$
$$= \pm\frac{1-\cos x}{\sin x}$$
$$= \pm\frac{\sin x}{1+\cos x}$$