2008 BC Calculus Free Response Solutions

1 a) Let $f(x) = \sin(\pi x)$, and $g(x) = x^3 - 4x$

$$A = \int_0^2 \left(f(x) - g(x)\right)dx = 4$$

b) $g(x) = -2$ at $a = 0.5391889$ and $b = 1.6751309$

$$A = \int_a^b \left(-2 - g(x)\right)dx$$

c) $$V = \int_0^2 \left(f(x) - g(x)\right)^2 dx \approx 9.978$$

d) $$V = \int_0^2 \left(\left(f(x) - g(x)\right)(3 - x)\right)dx \approx 8.370$$

2 a) $$L'(5.5) \approx \frac{L(7) - L(4)3}{7 - 4} = \frac{150 - 126}{3} = 8 \text{ people per hour}$$

b) $$\text{Ave } L(t) = \frac{1}{4}\int_0^4 L(t)\,dt$$

$$= \frac{1}{4}\left[\frac{276}{2} + \frac{332}{2}(2) + \frac{302}{2}\right]$$

$$= \frac{1}{4}\left[621\right]$$

$=155.25$ people

c) 3

L(t) is continuous and differentiable.

Since $L(3) > L(1)$ and $L(3) > L(4)$, there is a maximum for some t in (1, 4). At this t, $L'(t) = 0$.

Similarly, L attains a minimum on (3, 7) and a maximum on (4, 8).

d) $$\int_0^3 550te^{-t/2}dt \approx 972.784$$

$\therefore \approx 973$ tickets by 3 p.m.

3 a) $P_1(x) = 80 + 128(x - 2)$, so $h(1.9) \approx P_1(1.9) = 67.2$

$P_1(1.9) < h(1.9)$ since h' is increasing on the interval $1 \le x \le 3$.

b) $$P_3(x) = 80 + 128(x - 2) + \frac{488}{6}(x - 2)^2 + \frac{448}{18}(x - 2)^3$$

$h(1.9) \approx P_3(1.9) = 67.988$

c) The fourth derivative of h is increasing on the interval $1 \le x \le 3$, so $\max\limits_{1.9 \le x \le 2}\left|h^{(4)}x\right| = \frac{584}{9}$.

Therefore, $$\left|h(1.9) - P_3(1.9)\right| \le \frac{584}{9}\frac{\left|1.9 - 2\right|^4}{4!}$$

$$= 2.7037 \times 10^{-4}$$

$$< 3 \times 10^{-4}$$

4 a) $x(3) = -10$

Because $v(t) > 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, $t = 3$ and $t = 6$ are relative minimums.

$$x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$$

$$x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$$

b) 3

$x(0) = -2$

$x(3) = -10$ so $v(t)$ has to equal -8 for $0 < t < 3$

$x(5) = -7$ so $v(t)$ has to equal -8 for $3 < t < 5$

$x(6) = -9$ so $v(t)$ has to equal -8 for $5 < t < 6$

c) decreasing because $v(t) < 0$ and increasing

d) $v'(t) < 0$ for $0 < t < 1$ and $4 < t < 6$ because $v(t)$ is decreasing on these intervals

5 a) $f'(x) < 0$ for $0 < x < 3$ and $f'(x) > 0$ for $x > 3$

Therefore, f has a relative minimum at $x = 3$.

$$f''(x) = e^x + (x-3)e^x = (x-2)e^x$$

b) $f''(x) > 0$ for $x > 2$

$f'(x) < 0$ for $0 < x < 3$

Therefore, the graph of f is both decreasing and concave up on the interval $2 < x < 3$.

c) $$f(3) = f(1) + \int_1^3 f'(x)\,dx = 7 + \int_1^3 (x-3)e^x\,dx$$

$u = x - 3 \qquad v = e^x$

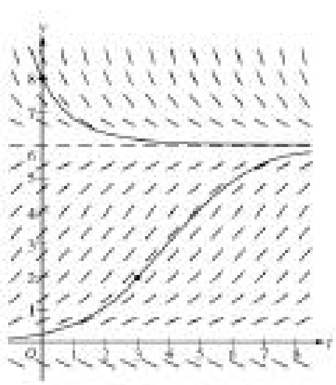
$du = dx \qquad dv = e^x dx$

$$f(3) = 7 + \left[(x-3)e^x - \int e^x\,dx\right]_1^3$$

$$= 7 + \left[(x-3)e^x - e^x\right]_1^3$$

$$= 7 + 3e - e^3$$

6 a)



b) $$f\left(\frac{1}{2}\right) \approx 8 + (-2)\left(\frac{1}{2}\right) = 7$$

$$f(1) \approx 7 + \left(-\frac{7}{8}\right)\left(\frac{1}{2}\right) = \frac{105}{16}$$

c) $$\frac{d^2y}{dx^2} = \frac{1}{8}\frac{dy}{dt}(6-y) + \frac{y}{8}\left(-\frac{dy}{dt}\right)$$

$$f(0) = 8,\ f'(0) = -2,\ f''(0) = \frac{5}{2}$$

$$P_2(t) = 8 - 2t + \frac{5}{4}t^2$$

$$f(1) \approx P_2(1) = \frac{29}{4}$$

d) The range of f for $t \ge 0$ is $6 < y \le 8$.