2009 AB Calculus Free Response Solutions

1. (a) $a(7.5) = v'(7.5) = \dfrac{v(8) - v(7)}{8 - 7} = -0.1$ miles/minute^2

(b) $\int_0^{12} |v(t)|\, dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t = 0$ to $t = 12$.

$$\int_0^{12} |v(t)|\, dt = \int_0^{2} v(t)\, dt - \int_2^{4} v(t)\, dt + \int_4^{12} v(t)\, dt$$
$$= 0.2 + 0.2 + 1.4 = 1.8 \text{ miles}$$

(c) Caren turns around to go back home at time $t = 2$ minutes. This is the time at which her velocity changes from positive to negative.

(d) $\int_0^{12} w(t)\, dt = 1.6$; Larry lives 1.6 miles from school.

$\int_0^{12} v(t)\, dt = 1.4$; Caren lives 1.4 miles from school.

Therefore, Caren lives closer to school.

2. (a) $\int_0^{2} R(t)\, dt = 980$ people

(b) $R'(t) = 0$ when $t = 0$ and $t = 1.36296$

The maximum rate may occur at 0, $a = 1.36296$, or 2.

$R(0) = 0$

$R(a) = 854.527$

$R(2) = 120$

The maximum rate occurs when $t = 1.362$ or 1.363.

(c) $w(2) - w(1) = \int_1^{2} w'(t)\, dt = \int_1^{2} (2 - t) R(t)\, dt = 387.5$

The total wait time for those who enter the auditorium after time $t = 1$ is 387.5 hours.

(d) $\dfrac{1}{980} w(2) = \dfrac{1}{980} \int_0^{2} (2 - t) R(t)\, dt = 0.77551$

On average, a person waits 0.775 or 0.776 hour.

3. (a) Profit $= 120 \cdot 25 - \int_0^{25} 6\sqrt{x}\, dx = 2500$ dollars

(b) $\int_{25}^{30} 6\sqrt{x}\, dx$ is the difference in cost, in dollars, of producing a cable of length 30 meters and a cable of length 25 meters.

(c) Profit $= 120k - \int_0^{k} 6\sqrt{x}\, dx$ dollars

(d) Let $P(k)$ be the profit for a cable of length k.
$P'(k) = 120 - 6\sqrt{k} = 0$ when $k = 400$.
This is the only critical point for P, and P' changes from positive to negative at $k = 400$.
Therefore, the maximum profit is $P(400) = 16{,}000$ dollars.

4. (a) Area $= \int_0^2 \left(2x - x^2\right) dx$

$$= x^2 - \frac{1}{3}x^3 \Big|_{x=0}^{x=2}$$

$$= \frac{4}{3}$$

(b) Volume $= \int_0^2 \sin\left(\frac{\pi}{2}x\right) dx$

$$= -\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Big|_{x=0}^{x=2}$$

$$= \frac{4}{\pi}$$

(c) Volume $= \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$

5. (a) $f'(4) \approx \dfrac{f(5) - f(3)}{5 - 3} = -3$

(b) $\int_2^{13} (3 - 5f'(x))\, dx = \int_2^{13} 3\, dx - 5\int_2^{13} f'(x)\, dx$

$$= 3(13 - 2) - 5(f(13) - f(2)) = 8$$

(c) $\int_2^{13} f(x)\, dx \approx f(2)(3 - 2) + f(3)(5 - 3)$

$$+ f(5)(8 - 5) + f(8)(13 - 8) = 18$$

5.

(d) An equation for the tangent line is $y = -2 + 3(x - 5)$.
Since $f''(x) < 0$ for all x in the interval $5 \le x \le 8$, the line tangent to the graph of $y = f(x)$ at $x = 5$ lies above the graph for all x in the interval $5 < x \le 8$.

Therefore, $f(7) \le -2 + 3 \cdot 2 = 4$.

An equation for the secant line is $y = -2 + \frac{5}{3}(x - 5)$.
Since $f''(x) < 0$ for all x in the interval $5 \le x \le 8$, the secant line connecting $(5, f(5))$ and $(8, f(8))$ lies below the graph of $y = f(x)$ for all x in the interval $5 < x < 8$.

Therefore, $f(7) \ge -2 + \frac{5}{3} \cdot 2 = \frac{4}{3}$.

6.

(a) f' changes from decreasing to increasing at $x = -2$ and from increasing to decreasing at $x = 0$. Therefore, the graph of f has points of inflection at $x = -2$ and $x = 0$.

(b) $f(-4) = 5 + \int_0^{-4} g(x)\,dx$

$= 5 - (8 - 2\pi) = 2\pi - 3$

$f(4) = 5 + \int_0^4 \left(5e^{-x/3} - 3\right) dx$

$= 5 + \left(-15e^{-x/3} - 3x\right)\Big|_{x=0}^{x=4}$

$= 8 - 15e^{-4/3}$

(c) Since $f'(x) > 0$ on the intervals $-4 < x < -2$ and $-2 < x < 3\ln\left(\frac{5}{3}\right)$, f is increasing on the interval $-4 \le x \le 3\ln\left(\frac{5}{3}\right)$.

Since $f'(x) < 0$ on the interval $3\ln\left(\frac{5}{3}\right) < x < 4$, f is decreasing on the interval $3\ln\left(\frac{5}{3}\right) \le x \le 4$.

Therefore, f has an absolute maximum at $x = 3\ln\left(\frac{5}{3}\right)$.