Another important limit

Find the value of y:

$$y = \lim_{h\to 0}\left(1+\frac{h}{x}\right)^{x/h}$$ (problem)

$$\ln y = \lim_{h\to 0}\frac{x}{h}\ln\left(1+\frac{h}{x}\right)$$ (because there is a variable exponent take the ln of each side)

$$\ln y = \lim_{h\to 0}\frac{\ln\left(1+\frac{h}{x}\right)}{\frac{h}{x}}$$ (convert to a L'Hopitals indeterminant form)

$$\ln y = \lim_{h\to 0}\frac{\dfrac{\frac{1}{x}}{1+\frac{h}{x}}}{\frac{1}{x}}$$ (L'Hopitals rule)

$$\ln y = \lim_{h\to 0}\frac{\frac{1}{x+h}}{\frac{1}{x}}$$ (simplify)

$$\ln y = \lim_{h\to 0}\frac{x}{x+h}$$ (simplify)

$\ln y = 1$ (evaluate the limit)

$y = e$ (change ln equation to exponential form)

$$\therefore \quad \lim_{h\to 0}\left(1+\frac{h}{x}\right)^{x/h} = e$$