

An Important Definition of e

Find the value of y:

$$y = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x \quad (\text{problem})$$

$$\ln y = \lim_{x \rightarrow \infty} x \ln \left(1 + \frac{1}{x}\right) \quad (\text{because there is a variable exponent take the ln of each side})$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{1}{x}\right)}{x^{-1}} \quad (\text{convert to a L'Hopitals indeterminate form})$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{\frac{-x^{-2}}{1 + \frac{1}{x}}}{-x^{-2}} \quad (\text{L'Hopitals rule})$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{-x^{-2}}{1 + \frac{1}{x}} \cdot \frac{1}{-x^{-2}} \quad (\text{invert and multiply})$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{x}{x+1} \quad (\text{simplify})$$

$$\ln y = 1 \quad (\text{evaluate the limit})$$

$$y = e \quad (\text{change ln equation to exponential form})$$

$$\therefore \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$