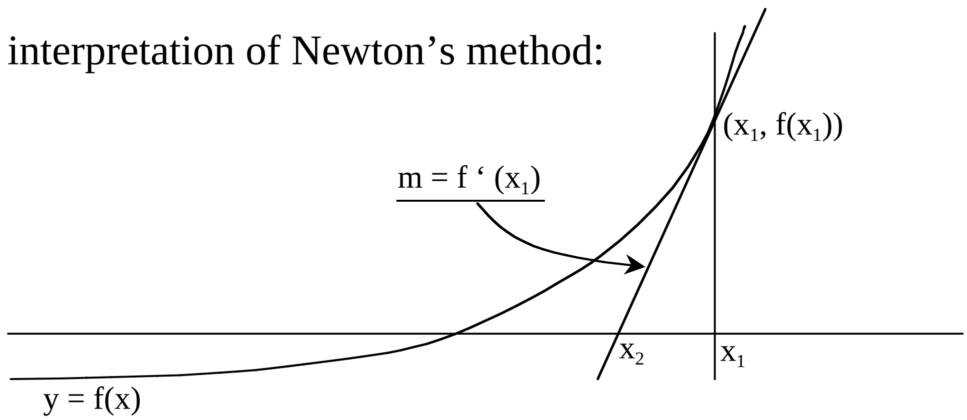
Newton's method

Newton's method is an iterative, calculus based, mathematical procedure that is used to find successive approximations (each one being more accurate than the previous) to the zero of a function.

Geometric interpretation of Newton's method:



The above diagram shows a function, y = f(x), that has a zero. Newton's method starts with an approximation to the zero, called x_1 in this case. On the function is a point, $(x_1, f(x_1))$, that corresponds to the x value called x_1 (the first approximation to the zero). At the point $(x_1, f(x_1))$ a line is drawn that is tangent to the function y = f(x). Because the slope of the tangent line is $f\,'(x_1)$, and the line passes through the point $(x_1, f(x_1))$, its equation can be found using the point slope formula of a line. The zero of this line, whose x value is called x_2, is a better approximation of the zero of the function. This process can be repeated as many times as desired to find the zero to the original function with as much accuracy as needed. Below, this process is developed mathematically.

The line with a slope of $f\,'(x_1)$ through the point $(x_1, f(x_1))$:

$y - f(x_1) = f\,'(x_1)\,(x - x_1)$ (equation using point-slope formula)

$(x - x_1) = \dfrac{y - f(x_1)}{f \copyright (x_1)}$ (divide by $f\,'(x_1)$)

$$x = x_1 + \frac{y - f(x_1)}{f'(x_1)}$$ (add x_1 to both sides of the equation)

$$x = x_1 - \frac{f(x_1)}{f'(x_1)}$$ (the zero of this line has a y value of zero)

This value of x now corresponds to x_2 in the above diagram.

Newton's method:	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Newton's method can be used quite nicely in the immediate mode of a graphing calculator:

- Place the function in Y_1
- Place the numerical derivative of Y_1 in Y_2
- On the home screen store the value of x_1 (the first approximation) in the variable x.
- Enter the Newton's method calculation, and store this is x.
- Press ENTER as many times as desire to obtain the required accuracy.

Y_1 = ________________

Y_2 = nDeriv(Y_1, X, X)

$x_1 \rightarrow X$

$X - (Y_1 / Y_2) \rightarrow X$

ENTER

ENTER

ENTER . . . .