2008 AB Calculus Free Response Solutions

1 a) Let $f(x) = \sin(\pi x)$, and $g(x) = x^3 - 4x$

$A = \int_0^2 \left(f(x) - g(x)\right)dx = 4$

b) $g(x) = -2$ at $a = 0.5391889$ and $b = 1.6751309$

$A = \int_a^b \left(-2 - g(x)\right)dx$

c) $V = \int_0^2 \left(f(x) - g(x)\right)^2 dx \approx 9.978$

d) $V = \int_0^2 \left(\left(f(x) - g(x)\right)(3 - x)\right)dx \approx 8.370$

2 a) $L'(5.5) \approx \frac{L(7) - L(4)3}{7-4} = \frac{150-126}{3} = 8$ people per hour

b) $\text{Ave } L(t) = \frac{1}{4}\int_0^4 L(t)dt$

$= \frac{1}{4}\left[\frac{276}{2} + \frac{332}{2}(2) + \frac{302}{2}\right]$

$= \frac{1}{4}[621]$

=155.25 people

c) 3

L(t) is continuous and differentiable.

Since L(3) > L(1) and L(3) > L(4), there is a maximum for some t in (1, 4). At this t, L ' (t) = 0.

Similarly, L attains a minimum on (3, 7) and a maximum on (4, 8).

d) $\int_0^3 550te^{-t/2}dt \approx 972.784$

$\therefore \approx 973$ tickets by 3 p.m.

3 a) When r = 100 cm and h = 0.5 cm, $\frac{dV}{dt} = 2000\,cm^3/min$ and $\frac{dr}{dt} = 2.5\,cm/min$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + h2\pi r\frac{dr}{dt}$

$2000 = \pi 100^2 \frac{dh}{dt} + 0.5(2\pi)100(2.5)$

$\frac{dh}{dt} \approx 0.039$ cm / min

b) $\frac{dV}{dt} = 2000 - R(t)$, so $\frac{dV}{dt} = 0$ when $r(t) = 2000$ and $t = 25$

$\frac{dV}{dt} > 0$ for $0 < t < 25$, and $\frac{dV}{dt} < 0$ for $t > 25$, so $t = 25$ is a maximum

c) $V = 6000 + 25(2000) - \int_0^{25} R(t)\,dt$

4 a) $x(3) = -10$

Because $v(t) > 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, $t = 3$ and $t = 6$ are relative minimums.

$$x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$$

$$x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$$

b) 3

$x(0) = -2$

$x(3) = -10$ so $v(t)$ has to equal -8 for $0 < t < 3$

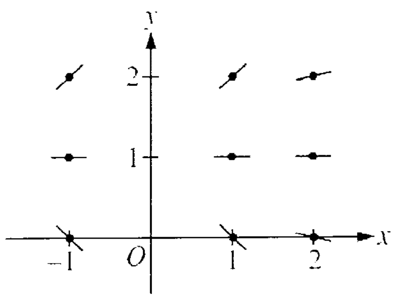
$x(5) = -7$ so $v(t)$ has to equal -8 for $3 < t < 5$

$x(6) = -9$ so $v(t)$ has to equal -8 for $5 < t < 6$

c) decreasing because $v(t) < 0$ and increasing

d) $v'(t) < 0$ for $0 < t < 1$ and $4 < t < 6$ because $v(t)$ is decreasing on these intervals

5 a)



b)

$$(y-1)^{-1}dy = x^{-2}dx$$

$$\ln|y-1| = -\frac{1}{x} + C$$

$$|y-1| = e^{-\frac{1}{x}+C}$$

$$|y-1| = e^{C}e^{-\frac{1}{x}}$$

$$-1 = ke^{-1/2}$$

$$k = -e^{1/2}$$

$$f(x) = 1 + -e^{1/2}e^{-1/x}$$

$$f(x) = 1 - e^{1/2-1/x}$$

c) $1 - \sqrt{e}$

6 a) $f\left(e^2\right) = \dfrac{2}{e^2}$ and $f'\left(e^2\right) = -e^{-4}$

$$y = -e^{-4}(x - e^2) + 2e^{-2}$$

b) $x = e$ is a relative maximum because $f'(e^-) > 0$ and $f'(e^+) < 0$

c) $$f''(x) = \frac{x^2\left(\dfrac{-1}{x}\right) - (1 - \ln x)2x}{x^4} = \frac{-3 + 2\ln x}{x^3}$$

$f''(x) = 0$ when $-3 + 2\ln x = 0$ and therefore $e^{3/2}$

d) $-\infty$ or D.N.E.