# 2007 AB Calculus Free Response Solutions

1 a) $A = 2\int_0^3 \left(\frac{20}{1+x^2} - 2\right)dx \approx 37.962$

b) $V = 2\pi\int_0^3 \left(\left(\frac{20}{1+x^2}\right)^2 - 2^2\right)dx \approx 1871.190$

c) $V = \frac{\pi}{4}\int_0^3 \left(\frac{20}{1+x^2} - 2\right)^2 dx \approx 174.268$

2 a) $\int_0^7 f(t)dt \approx 8264$ gallons

b) The amount of water in the tank is decreasing on the intervals $0 \le t \le 1.617$ and $3 \le t \le 5.076$ because $f(t) < g(t)$ for $0 \le t < 1.617$ and $3 < t < 5.076$.

c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0$, 3, and 7.
At $t = 0$, gallons = 5000.
At $t = 3$, gallons = $5000 + \int_0^3 f(t)dt - 250(3) \approx 5126.591$
At $t = 7$, gallons = $5126.591 + \int_3^7 f(t)dt - 2000(4) \approx 4513.807$
The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

3 a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$
$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$
Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r, $1 < r < 3$, such that $h(r) = -5$.

b) $\frac{h(3) - h(1)}{3-1} = \frac{-7-3}{3-1} = -5$
Since h is continuous and differentiable, by the Mean Value Theorem, there exists a value c, $1 < c < 3$, such that $h'(c) = -5$.

c) $w'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -2$

d) $g(1) = 2$, so $g^{-1}(2) = 1$.
$\left(g^{-1}\right)'(2) = \frac{1}{g'\left(g^{-1}(2)\right)} = \frac{1}{g'(1)} = \frac{1}{5}$. An equation of the tangent line is $y - 1 = \frac{1}{5}(x-2)$.

4 a) $x'(t) = 0$ at $t = \frac{\pi}{4}$, and $\frac{5\pi}{4}$

v(t) > 0 for t $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{5\pi}{4}, 2\pi\right)$ so x(t) is a minimum at t = 0, $\frac{5\pi}{4}$

x(0) = 0, $x\left(\frac{5\pi}{4}\right) < 0$, therefore the particle is farthest to the left when $t = \frac{5\pi}{4}$.

b) $x''(t) = e^{-t}(-\sin t - \cos t) - (\cos t - \sin t)e^{-t} = e^{-t}(-2\cos t)$

$Ae^{-t}(-2\cos t) + e^{-t}(\cos t - \sin t) + e^{-t}\sin t = 0$

$$A = \frac{1}{2}$$

5 a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8\text{ ft}$

Since the graph of r is concave down on the interval 5 < t < 5.4, this estimate is greater than r(5.4).

b) $\left.\frac{dV}{dt}\right|_{t=5} = 4\pi(30)^2 2 = 7200\pi\text{ ft}^3/\text{min}$

c) $\int_0^{12} r'(t)\,dt \approx 19.3\text{ft}$ $\int_0^{12} r'(t)\,dt$ is the change in the radius, in feet, from t = 0 to t = 12 minutes.

d) Since r is concave down, r' is decreasing on 0 < t < 12. Therefore, this approximation is less than $\int_0^{12} r'(t)\,dt$.

6 a) $f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}$ $f''(x) = -\frac{1}{4}kx^{-3/2} + x^{-2}$

b) k = 2 When k = 2, $f'(1) = 0$ and $f''(1) > 0$. f has a relative minimum value at x = 1 by the Second Derivative Test.

c) At this inflection point, $f''(x) = 0$ and $f(x) = 0$.

$f''(x) = 0$ $f(x) = 0$

$k = \frac{4}{\sqrt{x}}$ $k = \frac{\ln x}{\sqrt{x}}$

$$\frac{4}{\sqrt{x}} = \frac{\ln x}{\sqrt{x}}$$

$$k = \frac{4}{e^2}$$