BC Calculus
Review for Test # 6 (91–104)

91. A) Find $\lim_{x\to 0}(\csc x)x^2$ B) Find $\lim_{x\to 0^+}\left(\frac{1}{\sqrt{x}}-\frac{1}{x}\right)$.

92. Find $\left(f^{-1}\right)'(6)$ if $f(x)=x^3-2x+2$.

93. A) If $f(x)=x^2-5$, then f(2) = –1 and f(3) = 4. This means the function must have a root between x = 2 and x = 3. Use the average of these two numbers, 2.5, as the first approximation for Newton's methods and find the second approximation **without using a calculator**.

B) Find the positive root of $y=x^2-5$ to 9 decimal places by using Newton's method, and then check your answer by solving the equation.

C) Find the largest root of the equation $y = e^x - \sin x$.

94. Find the volume of the solid formed when the region bounded by $y=\cos x$ on the interval $[-\pi/2, \pi/2]$ is revolved around the line $x=\pi$.

95. Use the trapezoidal rule, with n = 4, to estimate the area under $y=\sin^{3/2} x$ on the interval [0, 3]. **Do not use a program**.

96. A) Find $\frac{dy}{dx}$ if $y=e^{|2x|}$. B) $\int_{-2}^{2}|x-1|\,dx$ = (Do not use a calculator.)

C) If f is a continuous function and f(3) = 12 is the maximum value and f(–2) = –6 is the minimum value of f then what can be said about:

C) the maximum value of $|f(x)|$?

D) the maximum value of $f|x|$?

E) the minimum value of $|f(x)|$?

F) the minimum value of $f|x|$?

97. The region bounded by $y = \tan x$ and the x axis, on the interval $[0, \pi/4]$, is the base of a solid. Every vertical cross section of the solid, perpendicular to the x axis, is a square. Find the volume of the solid.

98. A) $\frac{d}{dx}\left(\int_2^x \cos^{2/3} t\,dt\right)=$ B) $\int_1^7 \frac{1}{t}dt=$

99. A) Use differentials to estimate $(2.03)^2+(2.03)^3$.

B) Use the tangent line to estimate the value of $\log_2 8.5$.

100. A) $\int \tan x\, dx =$ B) $\int \tan^2 x\, dx =$

C) $\int \tan^3 x\, dx =$ D) $\int \tan^4 x\, dx =$

E) $\int \sec x\, dx =$ F) $\int \sec^2 x\, dx =$

101. A) Without using L'Hôpital's rule, prove that $\lim_{x\to 0} \frac{\sin x}{x} = 1$.

Evaluate the limits in Parts B and C.

B) $\lim_{x\to 0} \frac{\sin(13x)}{3x}$ C) $\lim_{x\to 0} \frac{127x}{\sin(81x)}$

D) Without using L'Hôpital's rule, prove that $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$.

E) Prove that $\frac{d}{dx}\sin x = \cos x$.

102. Evaluate the limits in Parts A, B, C and D.

A) $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$ B) $\lim_{h\to 0} \left(1+\frac{h}{x}\right)^{x/h}$ C) $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{4x}$ D) $\lim_{h\to 0} \left(1+\frac{h}{4}\right)^{4/h}$

E) Prove that $\frac{d}{dx}\ln x = \frac{1}{x}$.

F) Using the fact established in Part E, prove that $\frac{d}{dx}e^x = e^x$.

103. A) Write the epsilon-delta definition of limit, using as much as possible your own words.

B) Prove that $\lim_{x\to 3} (2x-3) = 3$.

C) If δ is as found in part B and it is desired that y be within 0.00001 of 3, where must x be?

D) Use your graphing calculator to prove that $\lim_{x\to 2} (2x^2 - 3x + 1) = 3$ by finding a δ which guarantees that y is within ε of 3, when $\varepsilon = 0.01$.

104. A) Draw a slope field for $\frac{dy}{dx} = -\frac{4x}{y}$.

B) Draw the particular solution to the differential equation in part (A) that passes through (1, 2).

C) Explain some of the basic characteristics of the slope field defined by dy/dx = 2x.

D) Explain some of the basic characteristics of the slope field defined by dy/dx = 2y.