Replacement Problems 132

132–18 Find the equation of the line tangent to the graph of $y = \sqrt[5]{x}$ at the point where x = 32. Use this equation to approximate $\sqrt[5]{30}$. Compare this result to the result you obtain in Problem # 24.

132–19 Find the equation of the line tangent to the graph of $y = x^2$ at the point where x = 1. Use this equation to approximate 1.1^2 and 1.2^2. Obviously we know these values to be 1.21, and 1.44. The approximations are close, but certainly not exact.

Let's think carefully about the process that was used to find the tangent line. We started with

$$y = x^2 \quad , \quad \frac{dy}{dx} = 2x, \quad x_0 = 1, \quad y_0 = 1$$

and proceeded to use point-slope form of the equation of a line to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$

In this case y_1, the y value we know, was called y_0, and x_1, the x value we know, was called x_0. Thus, y, the y value we seek, could be called y_1, and x, the x value we are evaluating at, could be called x_1. With all of this in mind, we might start with the point-slope equation

$$y_1 - y_0 = m(x_1 - x_0)$$

But the slope we use was found by evaluating the derivative of the equation, $\frac{dy}{dx}$, at x_0 and $x_1 - x_0$ could be called Δx. Thus, the point-slope equation is really

$$y_1 - y_0 = \left.\frac{dy}{dx}\right|_{x_0} (\Delta x)$$

Since we were really interested in computing y_1, let's solve this equation for y_1.

$$y_1 = y_0 + \left.\frac{dy}{dx}\right|_{x_0} (\Delta x)$$

Finally, let's generalize this equation:

$$y_n = y_{n-1} + \left.\frac{dy}{dx}\right|_{x_{n-1}} (\Delta x)$$

132–20 Begin with $y = x^2$, $x_0 = 1$, $y_0 = 1$, and the equation developed directly above to approximate 1.1^2, and 1.2^2. How do these values compare with the values computed in 132–19?